Reconstructing sea level from paleo and projected temperatures 200 to 2100AD

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Abstract

We use a physically plausible 4 parameter linear response equation to relate 2000 years of global temperatures and sea level. We estimate likelihood distributions of equation parameters using Monte Carlo inversion, which then allows visualization of past and future sea level scenarios. The model has good predictive power when calibrated on the pre-1990 period and validated against the high rates of sea level rise from the satellite altimetry. Future sea level is projected from IPCC temperature scenarios and past sea level from established multi-proxy reconstructions assuming that the established relationship between temperature and sea level holds from 200-2100 A.D. Over the last 2000 years minimum sea level (-19 to -26 cm) occurred around 1730 AD, maximum sea level (12 to 21 cm) around 1150 AD. Sea level 2090-2099 is projected to be 0.9 to 1.3 m for the A1B scenario, with low probability of the rise being within Intergovernmental Panel on Climate Change (IPCC) confidence limits.

1. Introduction

Rising sea level is probably the most important impact of anthropogenic climate change over the coming century. The approach used by IPCC (Meehl et al., 2007) to estimate future sea level rise has been to model the major components of sea level balance: thermal volumetric expansion and ice melting. IPCC AR4 estimates of sea level rise by 2100 are 18-59 cm (Meehl et al., 2007). However, these estimates have been challenged on the basis that large ice sheets appear to be changing much more rapidly (Ekström et al. 2006; Velicogna and Wahr, 2006) than models predict (Overpeck et al. 2006). A reality recognized by the IPCC summary report (IPCC, 2007). Small glaciers are well measured and understood and are likely to contribute only with 10-20 cm (Raper and Braithwaite, 2006) to 21st century sea level increase. Thermal expansion is also reasonably well understood (Domingues et al., 2008) and expected to contribute 10-30 cm (Bindoff et al., 2007). The large ice sheets are much more challenging both to measure and to model. Traditionally glaciological mass balance measurements are always difficult on large ice sheets because of the logistical problems associated with measuring snow accumulation and melt at representative points on the ice sheet, estimating iceberg calving, sub-glacial runoff and ice shelf basal melting. However, the older glaciological data with satellite and airborne radar-altimetry, and more recent GRACE data on ice sheet mass together show a trend towards increasingly negative mass balance for Greenland (Lemke et al., 2007).

Our theoretical understanding of the different contributors is incomplete as IPCC models underpredict rates of sea level rise 1993–2006 by ~40% (Rahmstorf et al., 2007). However, we know that the major contributors to sea level are all responding to changes in global temperature. A good approach is therefore to establish a semi-empirical model linking sea level rise to temperature. This allows us to provide model projections while evading the unknowns pertaining to individual contributors. This has previously been attempted by Rahmstorf (2007) assuming a linear relationship between the rate of sea level rise and temperature (Holgate et al., 2007), valid only for temperature-sea level response times of several centuries to millennia. However, earlier efforts indicated a much shorter lag (~20 years) between temperature and sea level rise (Gornitz et al., 1982) which taken together with the recent acceleration of the ice sheet contribution casts doubts on the assumptions in the Rahmstorf (2007) model.

In contrast to both Rahmstorf (2007) and Gornitz et al. (1982), we make use of much more than the historical record of sea level and temperature which spans at most the past 150 years. Historical evidence provides limits on sea level variability over the last few millennia (Sivan et al., 2004; Gehrels et al. 2005; Jansen et al., 2007). Sea level on glacial-interglacial timescales can be inferred from geologic evidence (Lambeck et al., 2004; Jansen et al., 2007). Past temperatures can be inferred from proxy data such as ice cores and tree rings (Jansen et al., 2007). This much longer set of

observational data allows us to construct a more advanced model of sea level response to changing temperature with 4 free parameters contrasting with only 2 or 3 parameters that were evaluated by previous authors (Rahmstorf, 2007; Gornitz et al., 1982). We employ a semi-empirical model linking sea level rise to temperature through a physically plausible but simple differential equation while explicitly calculating a response time. Our inversion approach ensures that the model is consistent with the rich long-term picture of sea level response to temperature. In addition we produce a new statistical methodology for evaluating the sea level data to avoid over-fitting.

The model is used to produce a reconstruction, with confidence limits, of past sea level over the past 2000 years that is consistent with all the proxy data used. The model parameters are validated against the rapid increase of sea level seen post 1990 by satellite altimetry. This model is then used with global temperature scenarios from IPCC to make predictions of sea level rise by the end of the 21st century.

2. Methods and Data

It is convenient to define both global mean temperature (T), and global mean eustatic sea level (S) relative to the mean over a well documented time interval. For clarity and ease of comparison we use 1980-1999 as the reference period for both S and T, following IPCC (Meehl et al., 2007). The ice masses on Earth and oceanic thermal expansion, the two major contributors to sea level rise, both respond to higher global surface temperatures by increasing sea level. On glacial time scales it is found that high sea level is associated with warm temperature (Jansen et al., 2007; Bintanja et al., 2005). It is reasonable to assume that there exists an equilibrium sea level (Seq) for a given temperature. This assumption does not exclude that changes in sea level might feedback into changes in temperature. The relationship between Seq and T must be non-linear for large changes in sea level and temperature, such as those that occur on glacial-interglacial timescales, and there may be multiple equilibria depending on the initial conditions. However, for the late Holocene-Anthropocene climate, where sea level is close to equilibrium and changes in sea level are much smaller (Lambeck et al. 2004; Jansen et al., 2007), we can linearize as:

$$S_{eq} = aT + b, \quad \text{(eqn.1)}$$

where a is the coefficient of sea level to a temperature change and b is a constant.

Changes in sea level are caused primarily by changes in global ice volume and global ocean heat content (Bindoff et al., 2007), both of which will have a response time to warming. Both ice melt and ocean warming will occur faster the further the system is from equilibrium. We therefore assume that sea level will approach S_{eq} with a characteristic response time (τ) as follows

$$\frac{\partial S}{\partial t} = (S_{eq} - S)/\tau, \quad \text{(eqn.2)}$$

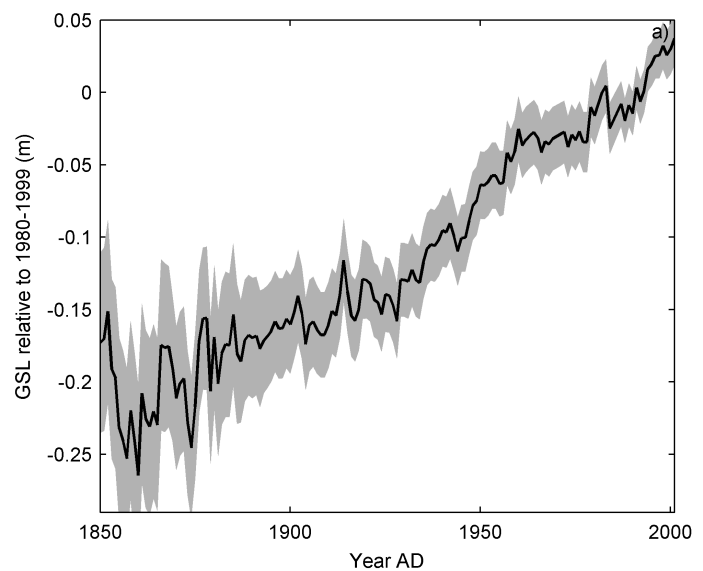where t is time. In reality each individual contributor (Gla-

Figure 1: Reconstructed Global Sea Level (Jevrejeva et al., 2006) (black line) and standard errors (dark grey shading).

ciers, Ice caps, the Greenland and Antarctic ice sheets, thermal expansion, etc) will have its own response time and that may even vary depending on the state of the system. E.g. it may be argued that ice sheet growth is a slow process whereas shrinkage is a fast dynamic process (Hansen, 2007). We will, therefore, restrict the use of equation 2 to a relatively short period dominated by sea level rise and we argue that we may approximate the system by a single effective response time. Equation (2) may be integrated to give sea level (S) over time using a history of T and knowledge of the initial sea level at the start of integration (S_0). The highly serially correlated uncertainties in observed sea level severely limit the number of free parameters it is possible to determine and thus the level of complexity in the model. For this reason we judge it impossible to split the model described by eqn. 1 and 2 into a sum of individual contributors without solid knowledge of many of the additional parameters introduced.

We use global surface temperatures (1850-2007) from HadCRUT3v (Brohan et al., 2006) as input to the above model and calibrate it against observed Global Sea Level (GSL) from tide gauges. The model is calibrated against the 'virtual station' GSL reconstruction (Jevrejeva et al. 2006) from 1850 to 2001, as it has published standard errors (see Fig. 1) and preserves volcanic signatures (Grinsted et al., 2007). This GSL reconstruction stacks 1023 tide gauge records using a stacking algorithm designed to minimize spatial bias while being independent of satellite altimetry.

The response time may be very long compared to the observational records of sea level and temperature. Hence, it may be difficult to determine the response time without any additional knowledge. We therefore extend the sea level and temperature variability much further back in time. To do this we extend the global HadCRUT3v record with Northern Hemisphere temperature reconstructions which span roughly 2 millennia. Rather than relying on published uncertainties in individual reconstructions, which do not have information on the serial correlation of errors, we use two very different Northern Hemisphere temperature reconstructions: The reconstruction from Moberg et al. (2005),

which has a pronounced Medieval Warm Period (MWP) and Little Ice Age (LIA), and the reconstruction from Jones and Mann (2004), which has much smaller MWP/LIA amplitude. Using these two proxies produces a much wider range of paleo-temperatures than would be the case from a simple consideration of errors in any particular reconstruction. These two reconstructions make use of different methods, contain different sets of proxy records within them, and both represent plausible scenarios of the past given the data available now. We also extend the GSL reconstruction prior to 1850 using the record of annual mean sea level from Amsterdam since 1700 (van Veen, 1945) correcting it for the post glacial land submergence rate of 0.16 mm yr^{-1} (Peltier, 2004). The issue of how well the Amsterdam record represents global sea level was treated in some detail in Jevrejeva et al. (2008b), and the representativity error dominates the uncertainties in local station vertical movement.

3. Experiments

We make several separate experiments using different data for the calibration:

1. 'Historical' using the HadCRUT3v temperatures, and post-1850 GSL only

2. 'Moberg' using the Moberg et al. reconstruction.

3. 'Jones & Mann' using the Jones & Mann (2004) reconstruction.

The Moberg and Jones & Mann experiments are calibrated against the extended GSL record using the long Amsterdam record. Additionally, we repeat the 3 experiments with the model calibration restricted to pre-1990 data and validate against the satellite derived sea level trend 1993-2006 (See validation section).

4. A priori constraints

The model parameters in equation 1 and 2 are unknown, but can be constrained by physicality. This aids model fitting by imposing a priori constraints on the model parameters.

1. Only positive response times are meaningful therefore we require $\tau > 0$.

2. Sea level in 1980-1999 was rising even though T=0 (by definition of the reference period) and we therefore know that $b > 0$ m (see eqn. 1).

3. In the Last Interglacial (LIG) temperatures inferred from deep ice cores were 3-5 °C warmer than present and sea level was 4-6 m higher (Jansen et al., 2007). It is therefore extremely unlikely that if future temperature remains below the LIG level that sea level would rise as much as then. Hence, we impose a weak constraint that no additional warming (T=0) results in a S_{eq} of less than 5 m (i.e. $b<5$ m).

4. Using equation 1 we get
   $a < (S_{max}-b_{min})/T_{min} < 2$ m/°C,
   where the maximum estimate of sea level (S_{max} = 6 m) and the corresponding minimal estimate of the temperature

difference (T_{min} =3 °C) were taken from LIG conditions (Jansen et al., 2007) and the minimal estimate of b_{min}=0 was taken from constraint 2. An alternative value for this constraint comes from Bintanja et al. (2005) who used a combination of observations and models for the sea level over the past million years, to conclude that sea level during glacial stages, air temperatures were ~17 °C lower than present, with a ~120 m sea level equivalent of continental ice present. These numbers give a much higher limit for a of 7 m/°C. We expect a to be positively correlated with total ice volume and the value that is applicable for the present ice sheet configuration is probably much smaller than that derived from glacial cycles. We weakly constrain a < 10 m/°C.

5. A naïve estimate of a can be obtained by considering that over the last 150 years there has been ~0.3 m of sea level rise and ~0.6 °C warming which gives ~0.5 m/°C. This must be considered a lower limit as sea level has not yet fully responded to the recent warming. The long term sea level rise from thermal expansion alone has been estimated to be ~0.5 m/°C (Meehl et al., 2007). The long term contribution from the ice sheets is potentially much greater (Meehl et al., 2007) and we therefore constrain $a > 0.5$ m/°C.

6. We start the integration of equation 2 at different times in the different experiments and the constraint we impose on S_0 must reflect that.

   a. For the integrations starting in 1850 S0 is constrained by the uncertainties of observed GSL and we apply the constraint that it was within 4 standard errors (=0.25 m) of -0.21 m.

   b. Mediterranean archaeological data (Sivan et al., 2004), and salt-marsh records from New England (Gehrels et al., 2005) suggest variations in sea level have not exceeded ±0.25 m from 2,000 to 100 yr before present. Globally sea level has been more stable over the last 3000 years than during much of Holocene, with sea level 2000 BP probably slightly lower than at present, but within 1 m of present day levels (Lambeck et al. 2004). So we use a weak constraint of $|S_0|< 1$ m for the 'Moberg' and 'Jones & Mann' experiments.

5. Inversion and GSL Uncertainties

The model described by equations 1 and 2 allows us to calculate sea level using observed temperatures. To find the model parameters which fit observed sea level we use an inversion scheme from Mosegaard and Tarantola (2002) that is similar to the simulated annealing (Kirkpatrick et al., 1983) approach. This scheme, called inverse Monte Carlo, has the advantage that it produces the statistical distribution of the model parameters rather than just minimizing the misfit. We can evaluate the likelihood of a given parameter set ($m=[\log(\tau),a,b,S_0]$) by calculating the misfit between observed and modelled sea level. By calculating the misfit to sea level rather than the sea level rate we avoid the need to smooth the sea level data to reduce the noise as done by
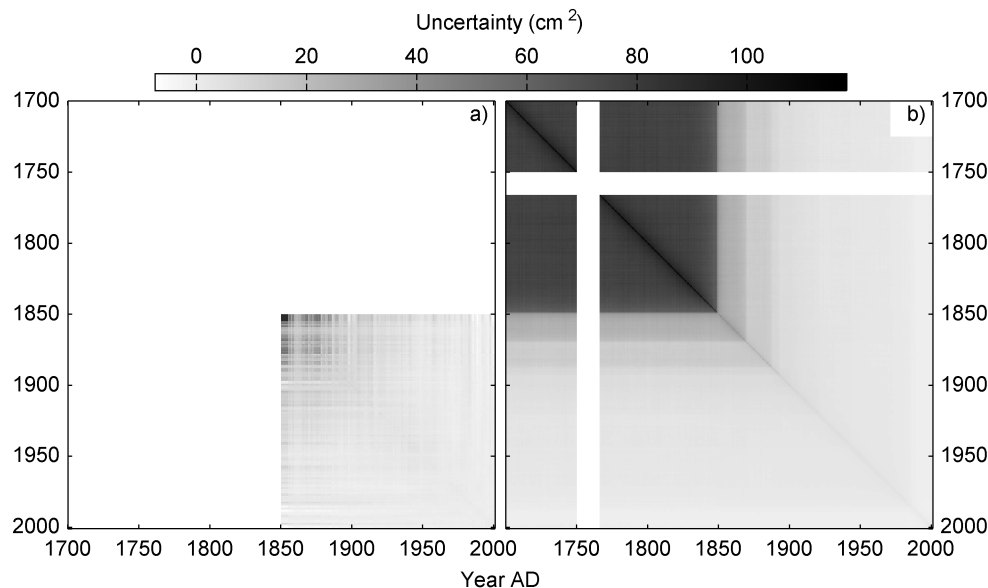
Figure 2: Two estimates of the GSL uncertainty covariance matrix (C in eq. 3). The leading diagonal show the squared standard errors conventionally plotted as a confidence interval on GSL. a) Estimate based on jack-knifing individual regions. b) Parametric Monte Carlo estimate. The pre-1850 values describe the uncertainty associated with using the Amsterdam record to extend GSL. The white bars are a data gap in the Amsterdam record. Both C matrices have very similar leading diagonal and off-diagonal magnitude and structure

Rahmstorf (2007). Smoothing the data reduces the degrees of freedom which invalidates traditional methods assessing the significance of the fit (Holgate et al., 2007; Schmith et al. 2007). We define the likelihood function following Mosegaard and Tarantola (2002) as

$$L(m) = ke^{-\frac{1}{2}(S(m)-S_{obs})^{T}\mathbf{C}^{-1}(S(m)-S_{obs})} \text{ , (eqn.3)}$$

where k is a normalization constant, S_{obs} and $S(m)$ are the vectors of observed and modelled sea level respectively, ^{T} denotes transpose, and C is the uncertainty covariance matrix where C_{ij} is the covariance between the GSL uncertainty at time-instants i and j. When **C** is a diagonal matrix, the exponent of eqn. 3 reduces to the traditional squared deviation appropriate for independent data and the maximum likelihood model will therefore correspond to the least squares fit. The off diagonal elements account for the fact that the uncertainties are not independent in time but are in fact highly correlated. Wunsch et al. (2007) show that even on decadal scales systematic (or dependent/correlated) errors are likely to dominate most estimates of GSL rise. Describing the time dependence of errors through a **C** matrix is a much more accurate and rich representation than simply reducing the 'effective degrees of freedom' based on e.g. the properties of a red noise process, or by doing an Empirical Orthogonal Function (EOF) analysis or fitting a reduced auto-regressive model. For example a time series contaminated with simple red noise, end point time series values constrain the model more than a central point, because the uncertainties of the central point are shared with the neighbors to both sides and this is contained in their **C** matrix values. The evaluation of the **C** matrix is therefore important to ensure that the data are not over-fitted due to the very high and time varying autocorrelation structure of the uncertainties.

The structure of **C** is rather complex. E.g. any errors in the

GSL reference period will be anti-correlated with those from other periods, simply because the reference period mean is subtracted from the other data. To estimate the **C** matrix it is useful to consider the steps involved in the reconstruction of GSL which are described in Jevrejeva et al. (2006) and Grinsted et al. (2007). First each station is assigned to one of 13 regions (Jevrejeva et al., 2006; Grinsted et al., 2007). We then recursively collapse the two closest stations within a region (by averaging their rates of sea level rise) into a new virtual station half-way between them until only one station remains. This last remaining virtual station represents the average for the entire region. This ensures that isolated tide gauge records are given more weight. We calculate GSL by integrating the rate of change in GSL (dGSL), with the dGSL curve as the arithmetic average of the sea level rates for the following regions: Northeast Pacific, Southeast Pacific, West Pacific, Central Pacific, Indian, Arctic, Antarctic, Mediterranean, Northeast Atlantic, Northwest Atlantic, Southeast Atlantic and Southwest Atlantic. A comparison of the resulting GSL reconstruction with other reconstructions can be found in the supplementary information of Grinsted et al. (2007). The errors in GSL are due to the errors in the regional rate series and the changing global coverage.

There is large spatial coherence in the representativity error of tide gauges (the difference between sea level at a tide gauge and GSL). We therefore estimate the errors from the set of regional sea level records, rather than individual tide gauges. The simple way of estimating the **C** matrix is by
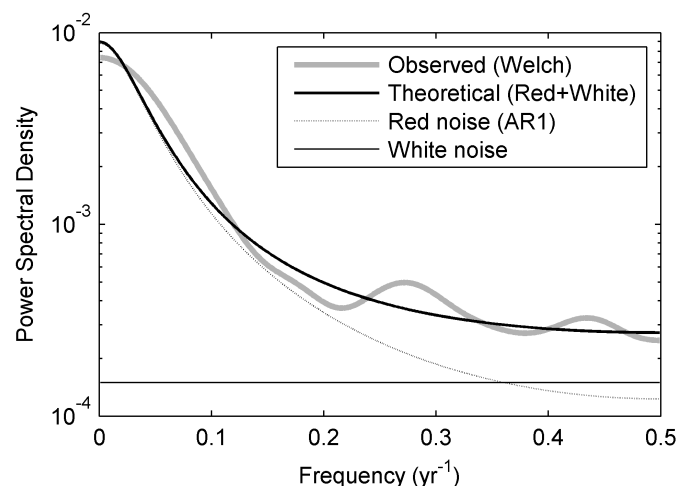

Figure 3: Noise spectra of NE Atlantic region sea level residuals (thick gray line) and the synthetic spectrum (thick black) from the sum of a red noise (dotted) and white noise (thin black) process.
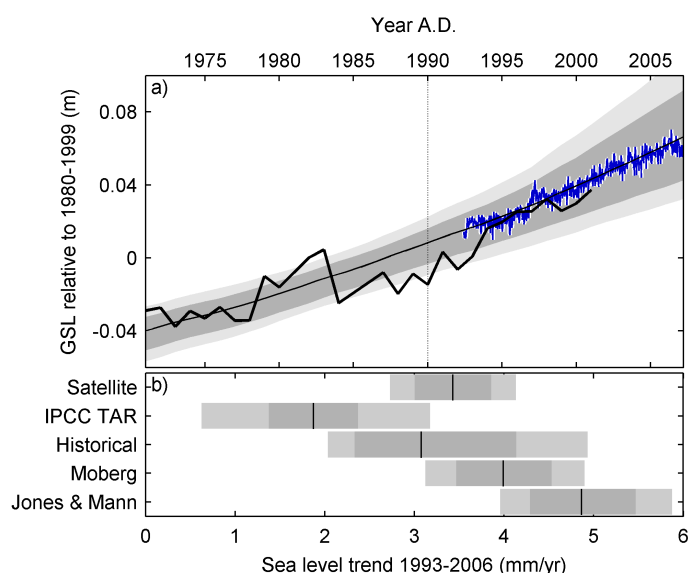
Figure 4: Validation of modeled sea level against Topex/Poseidon satellite altimetry. Model calibration was restricted to pre-1990 GSL data only. a) Validation of the historical experiment showing: Median model (thin black line), one standard deviation (dark grey band), 5-95 percentiles (light grey band), reconstructed global sea level (thick black curve, as Fig 1; Jevrejeva et al., 2006) and satellite derived sea level (blue; Leuliette et al., 2004). b) Comparison of the linear trend for the 1993-2006 period from satellite observations and models. Shading has same meaning in a) and b) except for IPCC TAR (Church et al., 2001) trends which shows the average (thin black line), range of all models (dark grey), range of all models including uncertainties in land-ice changes, permafrost changes and sediment deposition (light gray).

jackknifing the regions (Miller 1968) and thus getting 11 alternative GSL reconstructions. The **C** matrix can be calculated from the auto-covariance of the residuals to the full GSL reconstruction (Fig. 2a).

However, there are too few regions to get a robust jackknife estimate of **C**. For example, we are skeptical of the apparent relatively low uncertainty in 1870 compared to 1900. Instead, we obtain a parametric Monte Carlo estimate by simulating the time-varying noise spectra of regional sea level where we can use the overall shape of the jackknife estimate as a validation. The noise characteristics are determined from the difference between regional sea level rate and the dGSL reconstruction. A typical noise spectrum can be approximated by the summation of a red noise and a white noise process (Fig. 3).

The noise parameters for each region are estimated by minimizing the squared residuals between the observed spectrum (estimated using the Welch method) and the theoretical spectrum. We interpret the red noise as the difference between regional sea level and true GSL and the white noise represents the errors in the estimate of regional sea level. The red noise component is unchanging in time, while the white noise component is changing due to varying station coverage within the region. We estimate the time dependent noise variance as the moving variance in a 31 year wide window. The window width was chosen as a trade-off between high time res-

olution and a having a variance estimate with small errors. In practice we increased the window length until the general features of the variance curve were in agreement with the time varying station coverage. This set of noise parameters allows us to generate noise surrogates of regional sea level rate and therefore also of the noise in GSL. The **C** matrix is the average noise auto-covariance matrix of 5000 Monte Carlo simulations.

The parametric Monte Carlo method also allows us to usefully extend the **C** matrix for the period covered by the long Amsterdam tide gauge record only. The noise parameters needed to extend the **C** matrix are taken to be the same as the earliest part of the North-East Atlantic regional record as this is period is only based on a single tide gauge. The resulting Monte Carlo estimate of the extended C matrix is shown in Fig. 2b.

If the errors were well represented by a first order AR process (red noise) the **C** matrix would show a diagonal structure with variance falling with distance from the leading diagonal. However, the pattern shows a strong time dependence and the errors have a more 'rectangular' shape more consistent with a Markov chain random walk process. This is due to the accumulation of dGSL errors in the integrated GSL. Negative uncertainties indicate anti-correlation between errors in the reference period with other errors.

We use 2 000 000 member ensemble Monte Carlo inversion (Mosegaard and Tarantola, 2002) to sample the model space according to the likelihood probability while honouring the a priori knowledge. This allows us to estimate the likelihood distribution of both model parameters and modelled sea level.

6. Validation

To test the power of the model we need to employ a calibration and verification period using the historical sea level record. The post-1990 satellite epoch has previously been used as a test of models of sea level rise, assuming that satellites represent a much truer realization of GSL than tide gauges. This relatively short period is a stiff test as sea level rose at 3.1 mm/yr (1993-2003, Bindoff et al., 2007) compared with the 20th century mean rate of 1.8 mm/yr (Jevrejeva et al., 2006). Rahmstorf et al. (2007) find that IPCC model projections under-predict satellite rates by about 40% (Rahmstorf et al. 2007). We repeat this exercise and restrict the model calibration to pre-1990 data for all 3 model experiments and validate their projections against the observed satellite altimetry GSL (Leuliette et al., 2004) from 1993 to 2006. Figure 4 shows that the predictions from the Moberg

Table 1: Model parameters

	τ (yr)	a (m/°C)	b (m)	S_0 (m)	$T|_{S=0}$ (°C)	a/τ (m/°C/yr)†
Moberg*	208±67	1.29±0.36	0.77±0.25	-0.002±0.55	-0.59±0.06	0.0063±0.0011
Jones & Mann#	317±117	2.56±0.90	1.36±0.52	-0.035±0.55	-0.53±0.03	0.0082±0.0011
Historical only	1193±501	3.10±1.64	3.68±1.15	-0.249±0.04	-0.99±1.06	0.0030±0.0018

* Using reconstructed temperatures since A.D. 0 (Moberg et al., 2005). # Using reconstructed Northern Hemisphere Temperatures since A.D. 200 (Jones and Mann, 2004). † Compare Rahmstorf (2007) value of 0.0034 m/°C/yr.
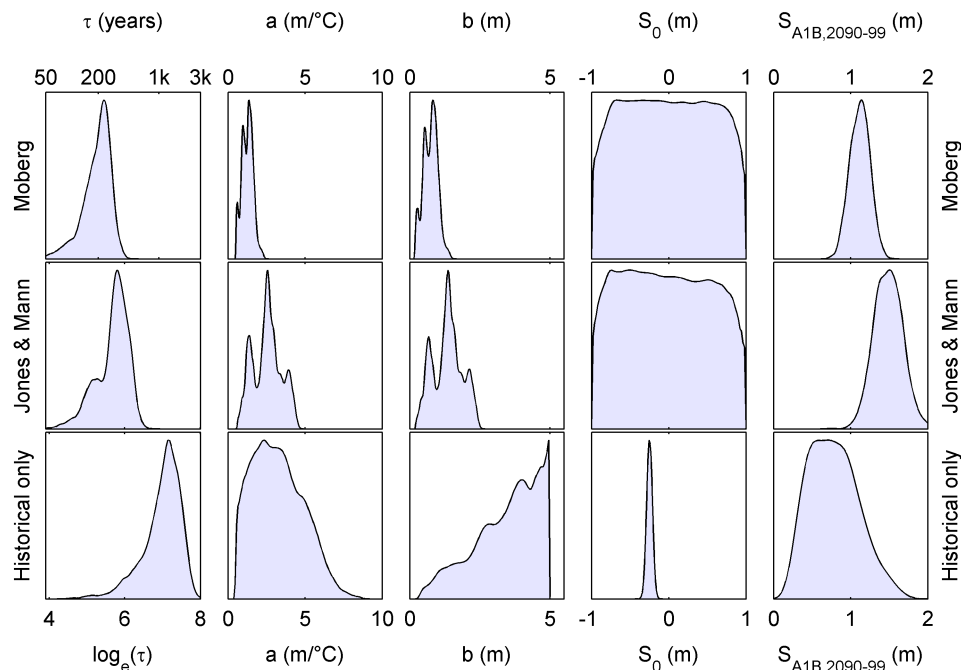
Figure 5: Empirical likelihood probability density functions of the 4 model parameters for each of the 3 experiments and the likelihood of the projected sea level response in 2090-99 using A1B temperatures.

and Historical experiments are consistent with the satellite record within uncertainties despite rates of rise in the satellite era being much greater than over the calibration period (Lombard et al., 2007). The experiment using the Jones & Mann reconstruction results in too high a rate of rise over the satellite period. We conclude that the estimated sea level rise from both the Moberg and Historical experiments is closer to reality than the IPCC estimates of sea level rise when tested over the same period (1993-2006).

7. Results and Discussion

The model parameters for the 4 inversion experiments (Moberg, Jones & Mann, Historical, and Validation) can be found in Table 1 and Fig. 5. The likelihood probability density of b for the 'Historical' experiment is cut-off very suddenly by the $b<5$ m constraint (Fig. 5 & 6) which we take as a sign that the model may be underdetermined without the use of additional data. The simple conclusion is that the calibration time series is too short relative to the response time. Inclusion of the additional pre-1850 data clearly favors faster response and a higher sensitivity ($a\tau^{-1}$) than instrumental observations alone (Table 1).

The response time for the 'Moberg' and 'Jones & Mann' inversions are 200-300 years. This is an order of magnitude faster than the response time found by examining sea level response over glacial-interglacial cycles or oceanographic time constants such as the 2-5 kyr lag between surface temperature and deep oceanic temperatures (Bintanja et al., 2005). E.g. Fitting an exponential ($k_1 e^{t/\tau}+k_2$) to Holocene sea level rise (10 kyr – 1 kyr BP) (Jansen et al., 2007) yields a response time of ~2500 years. Such an exponential is appropriate as Holocene sea level rise is primarily responding to a preceding large warming at the end of the glacial. The response time which applies today, however, may be very different from that which governed sea level rise throughout

most of the Holocene. In reality the temperature response of global sea level has many components, each with their own response time. When examining sea level throughout the whole Holocene only the long lived components (such
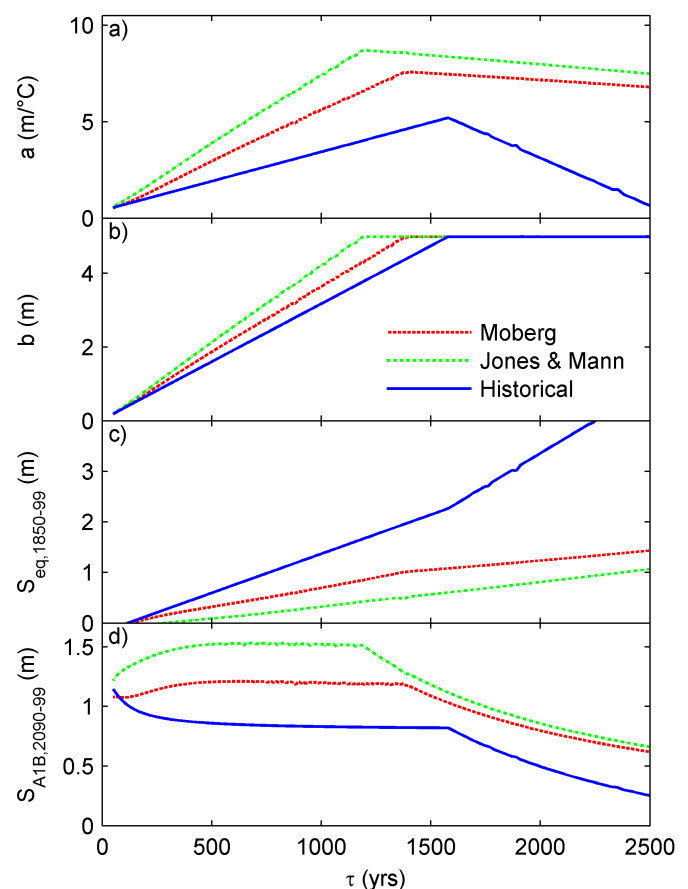

Figure 6: Best fitting model parameters and resulting sea level projections for prescribed response times. a) the sensitivity of equilibrium sea level to temperature change. b) equilibrium sea level rise for zero temperature change (relative to 1980-2000). c) the equilibrium sea level (eqn.1) for 1850-1899 temperatures ($T = $ -0.52 K). d) Resulting sea level projections for 2090-2099 using A1B temperatures (Meehl et al., 2007). The likelihood of different response times is shown in Fig. 4.

Table 2: Projected sea level rise 2090-2099 for the IPCC scenarios.

	A1B	A1FI	A1T	A2	B1	B2	T0*
Moberg	0.91-1.32	1.10-1.60	0.89-1.30	0.93-1.36	0.72-1.07	0.82-1.20	0.21-0.38
Jones & Mann	1.21-1.79	1.45-2.15	1.18-1.76	1.24-1.83	0.96-1.44	1.09-1.62	0.29-0.49
Historical only	0.32-1.34	0.34-1.59	0.32-1.32	0.32-1.37	0.30-1.10	0.31-1.22	0.22-0.44
Imm. / Inf.†	0.8 / 0.8	1.2 / 1.0	0.7 / 0.8	1.0 / 0.8	0.6 / 0.7	0.7 / 0.8	0.0 / 0.3
IPCC	0.21-0.48	0.26-0.59	0.20-0.45	0.23-0.51	0.18-0.38	0.20-0.43	

Range is 5-95 percentiles. * T0 is a scenario with no temperature rise. † Imm./Inf. refers to the projections assuming an immediate/infinite response time and with model parameters obtained from ordinary least squares (i.e. not using C).

as the isostatic rebound) with a slow response dominate because the fast components appear as noise only on the long term response. However, over the last few millennia sea level has almost equilibrated (Lambeck et al. 2004) to the ice age termination and the system is dominated by the fast response. Hence, millennial scale response times are inconsistent with the observed rise in sea level. Fig. 6 shows the quality of model parameters while prescribing the response time. From this we can conclude that the well-founded constraint that $b<5$ m obtained from sea level rise during the last interglacial (see a priori constaints section) effectively excludes response times longer than about 1500 years.

We can combine a and b to give the temperature which is needed to stop sea level from rising ($T|_{S=0}$, Table 1), which has values of about -0.6 °C for the experiments including the paleo temperature reconstructions. This indicates that temperatures around the beginning of the 20th century were close to long term equilibrium, consistent with geological evidence showing rates of sea level change during the last 3000 years were at Holocene lows (Lambeck et al. 2004; Gehrels et al. 2005; Sivan et al., 2004). For the same reason we consider it implausible that GSL was several meters below equilibrium at the end of the Little Ice Age (Fig. 5c). Hence, we conclude that millennial scale response times in the Historical experiment are also implausible.

It has been widely assumed that glaciers and thermal expansion are going to be the dominant contributors to 21st century sea level rise because they have a fast response time. IPCC estimate (Bindoff et al., 2007) that large ice sheets contribute 0.42 mm/yr to GSL since 1993, however more recent studies (Jevrejeva et al., 2008a; Lombard et al. 2007) suggest this is too low, and that ice sheets are likely contributing 0.86 mm/yr. Smaller glaciers contribute about 0.8 mm/yr (Bindoff et al., 2007), implying that since 1990 large ice sheets are dominating the mass contribution to GSL rise. This has prompted the concern that ice sheets may have a much faster response to warming than models predict (Hansen, 2007). We find that projected 21st century sea level is virtually independent of response time (Fig. 6 and table 3) and that IPCC model projections are much too low even for millennia scale response times (Table 2).

8. Sea level the past 2000 years

Modelled past sea level (Fig. 7) shows small variability, consistent with geological evidence (Lambeck et al. 2004; Gehrels et al. 2005). The two experiments 'Jones & Mann' and

'Moberg' have the same likelihood function and the relative likelihood of their fits can therefore be directly compared by examining the mean likelihood over all accepted models in the inverse Monte Carlo. We find that using the Moberg temperature reconstruction gives fits that on average have a factor 23 greater likelihood. We also note that the Jones & Mann experiment resulted in a much too high sea level rate when validated against satellite altimetry. Thus the Moberg et al. (2005) temperature reconstruction is relatively more consistent with the observed sea level record than that of Jones & Mann (2004). We therefore consider the results from the Moberg experiment to be best. The primary source of misfit of the 'Jones & Mann' experiment can be traced to too high GSL in the 18th century (fig. 7). The Jones and Mann reconstruction does not have a cold enough LIA to reproduce this low sea level without some long term memory of the initial sea level (S_0). The consequence on the likelihood density of S_0 can be seen in Fig. 5. The amplitude of the LIA temperature minimum is critical to ensure satisfactory model fits to the tide gauge record.

Robust findings are that reconstructed sea level shows a LIA minimum at ~1730 and a local MWP maximum at 1100-1200 (Fig. 7). The timing of maximal glacier extent during the LIA varied from region to region and even within the regions. For example many glaciers in the Americas were largest in 1700-1750 (Luckman and Villalba, 2001), whereas in the European Alps (Bradley 1999) it was rather earlier and in the Arctic somewhat later (Svendsen 1997). The sea level maximum during the MWP is 12 and 21 cm higher than the 1980-1999 average for the Jones and Mann (2004) and Moberg et al. (2005) respectively. The 12-21 cm higher sea level stand during the MWP is likely the highest sea level since the previous interglacial period 110 000 years ago, and was produced by an extended period of warming, allowing time for glaciers and thermal expansion to reach a climatic balance. Hence, the cooler than present temperatures in the MWP is consistent with higher than present sea level. Table 2 (T0) shows that the sea level at 2090-2099 will be higher than MWP even with no rise in temperatures above the present.

9. Sea level projections

We can project the response of sea level to future temperature scenarios assuming that the model parameters which we find for the past are applicable until 2100 AD. In sea level context 100 years must be considered the near term and extrapolation of the relationship is probably still reliable. However, we can not exclude the possibility that the linearity of eq. 1 breaks down in a warmer climate. Such non-linear conditions must have been prevailing during the rapid deglaciation ~14600 years ago (Meltwater pulse 1A) where sea level rose by ~20 m in less than 500 years (Weaver
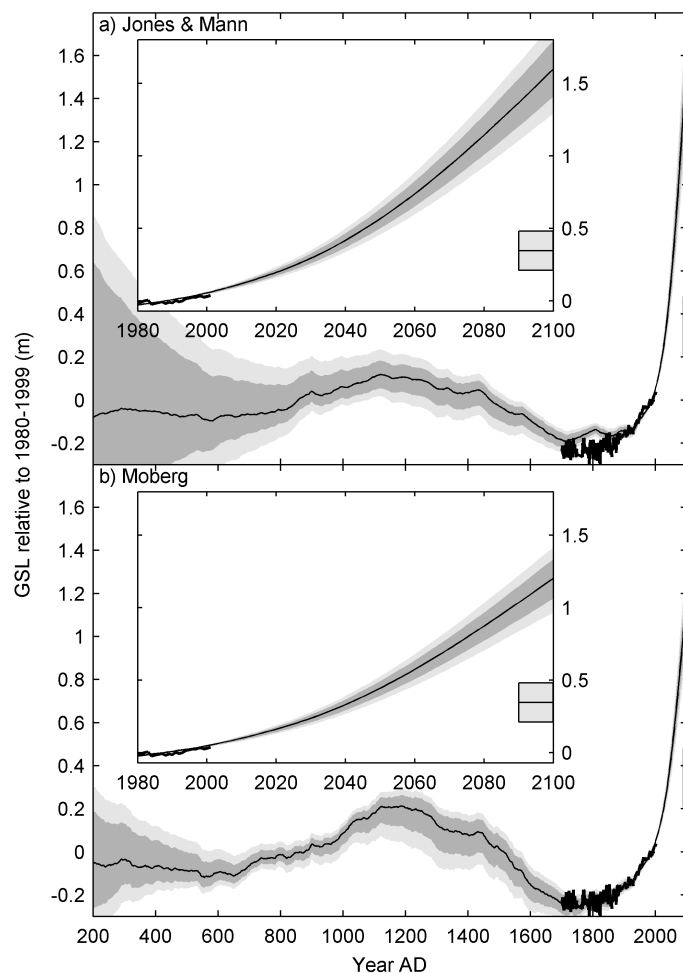
Figure 7: Projected sea level based on IPCC scenario A1B using temperature reconstructions of (a) Jones and Mann (2004) and (b) Moberg et al.(2005). Empirical likelihood distribution of sea level from 2 million inverse Monte Carlo ensemble. Thin black line: median, dark grey band: one standard deviation, light grey band 5-95 percentiles. Thick black line: reconstructed GSL (Jevrejeva et al., 2006) extended to 1700 using Amsterdam sea level (van Veen, 1945). Box: shows IPCC A1B estimates 2090-2100 (see table 2). Insets show the projections and fits to the GSL data in greater detail.

et al., 2003). By treating future temperatures as scenarios we effectively decouple them from the sea level model, and possible feedbacks that sea level may have on temperature are not taken into account. E.g. higher sea level is likely associated with greater ice loss which may influence global climate through albedo changes and changes to the fresh water flux to the world oceans.

We model future sea level rise using the IPCC AR4 (Meehl et al., 2007) projections of global mean surface temperature for 6 scenarios. Additionally we model the sea level response to a temperature scenario (T0) where T is kept constant at the 1980-1999 average. The IPCC confidence intervals fall well below the range of projections; Table 2 shows that all IPCC scenarios produce sea level rise about a factor of 3

Table 3: Correlation coefficients of accepted models in the 'Moberg' experiment.

	$\log(\tau)$	a	b	S0
a	0.89			
b	0.92	1.00		
S_0	-0.04	-0.05	-0.05	
$S_{A1B,2090-99}$*	0.10	0.47	0.40	-0.04

* the projected sea level in 2090-2099 using A1B temperatures.

smaller than our predictions, the insets in Fig. 7 show the projected sea level rise under the IPCC A1B temperature scenario. The projections in table 2 are largely independent of the response time (see Fig. 6 and table 3). We illustrate this robustness by comparing the projected sea level obtained assuming an immediate response time (as in Gornitz et al. 1982, but zero time lag), and an infinite response time (as in Rahmstorf, 2007). The projections resulting from these two simple models calibrated using naïve ordinary least squares agree qualitatively with the results from the 4 parameter model. The main difference between the projections from the 3 different inversion experiments (table 2) can be traced to the memory of LIA temperatures (Fig. 6d). The Moberg et al. (2005) reconstruction has a colder LIA than that of Jones & Mann (2004) and the Moberg experiment results in a lower sea level for the projection period. In the Historical experiment late 19th century sea level is much below equilibrium (Figure 6c) which suggests that it is coming out of an LIA which was colder than that in Moberg et al. (2005), and hence does not match any reconstructions of global paleo-temperature.

The striking difference between the IPCC AR4 projections and those presented here (table 2) naturally leads to the question: What process could possibly explain such a large difference? Our simple model does not allow us to attribute the sea level rise to individual contributors. We can, however, consider all the known major contributors and speculate as to which one is most likely. Models of global oceanic heat content show good agreement with observations (Domingues et al. 2008) and we therefore consider the thermal expansion contribution to be well modeled in IPCC AR4. The total volume of all Glaciers and Ice caps has been estimated to be 0.15-0.37 m sea level equivalent (Lemke et al., 2007) and can therefore not explain the difference. This leaves the contribution from ice sheets as the only major candidate. The surface mass balance of the ice sheets has been taken into account in the IPCC AR4 projections, whereas current ice sheet models do not represent rapid changes in ice flow and the dynamical contribution may therefore have been severely underestimated (Lemke et al., 2007). Thus we reason that the large projected sea level rise can most likely be ascribed to dynamical effects of the big ice sheets. Pfeffer et al. (2008) estimates the plausible range of the dynamical contributions and gives a best guess for the total sea level rise at 0.8 m and 2 m as an upper limit. This range is compatible with our projections. Thus ice sheet dynamical effects are the most likely source of discrepancy between our projections and those of the IPCC AR4.

The model parameters are determined empirically and it can therefore only model effects present in the calibration period. Thus, if we attribute a substantial part of the projected rise to ice sheet dynamics then we must ask ourselves if this contributor was active in the calibration period. Several studies have found accelerated ice discharge in the satellite observation period (Lemke et al., 2007), indicating that recently ice dynamics are contributing to sea level rise. We note that roughly 25% of the observed sea level trend over

the past 50 years can not be accounted for by the sum of all known contributors (Jevrejeva et al., 2008a; Bindoff et al., 2007). We speculate that dynamic ice loss may have played a greater role than generally appreciated in this period. We note that the rates of sea level rise in the 1950s were comparable to those we have had since the 1990s (Jevrejeva et al., 2006). However, we can not exclude that a completely unknown mechanism is responsible for both the past missing contributor, and the difference between our and the IPCC projections.

10. Conclusions

We approximate global sea level with a simple model forced by temperatures which we calibrate using inverse Monte Carlo methods. The model has good predictive power when calibrated on the pre-1990 period and validated against the high rates of sea level rise from the satellite altimetry. By including paleo-reconstructions of temperatures we produce the first well-constrained continuous sea level reconstruction for the last 2000 years. This indicates that present sea level is within ~20 cm of the highest level for 110 000 years. We show that post-1850 sea level rise can be approximated by models of both millennial- and century-scale response times. However, millennia scale response times imply implausible cold little ice age conditions, and are inconsistent with sea level observations 1700-1850. The inclusion of paleo-temperature reconstructions allows us to determine that present-day sea level rise is dominated by a fast 200-300 year response time to temperature (Table 1). We further find that the Moberg et al. (2005) temperature reconstruction is more consistent with observed sea level rise than the Jones and Mann (2004) reconstruction which we conclude does not have a cold enough Little Ice Age.

Having established models linking temperature to sea level rise, we project 21st century sea level using IPCC projections of temperature as forcing (Fig. 7, Table 2). We find that IPCC projections of sea level rise 2090-2099 are underestimated by roughly a factor 3 (Table 2). The likely rates of 21st century sea level rise far exceed anything seen in the last 2000 years. In comparison, the period 14000-7000 BP had an average rise rate of 11 mm yr-1 (Bard et al., 1996). This is similar to rates we predict by the 2050s. Rapid rates of sea level rise must be associated with decay of continental ice sheets. This interpretation is consistent with estimates of future sea level taking into account the plausible range of the ice sheet dynamical contribution (Pfeffer et al. 2008). This is consistent with observations of accelerating mass loss from Greenland (Overpeck et al. 2006), and possibly the West Antarctic Ice Sheet (Velicogna and Wahr, 2006). Even 1.5 m represents only 10-15% of the total ice volume in these particular ice sheets.

Acknowledgements

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Additional material

A version of Figure 7b better suited for presentations.

A change in mean sea level of one meter or more can increase the frequency of what otherwise would be characterized as extreme events by more than a factor 1000.
Image Credit: Northland Regional Council, New Zealand.